Skewed distributions fixed by diagonal partons at small x, ξ and $\gamma^* p \to Vp$ at HERA

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We show that the skewed parton distributions are completely determined at small x and ξ by the conventional diagonal partons. We study the application to diffractive vector meson production at HERA.

1. Introduction

Data are becoming available for processes which are described by off-diagonal (or skewed) parton distributions. A relevant example is diffractive vector meson production at HERA, $\gamma^* p \to Vp$ with $V = \rho, J/\psi$ or Υ, where at high $\gamma^* p$ c.m. energy, W, the cross section is dominated by the two-gluon exchange diagram

$$\frac{d\sigma}{dt}(\gamma^* p \to Vp)\bigg|_{t=0} = \ldots [x_2 g(x_1, x_2; \mu^2)]^2 \quad (1)$$

where g is the off-diagonal ($x_1 \neq x_2$) gluon distribution with

$$\begin{aligned} x_1 &= (Q^2 + M^2_{q\bar{q}})/W^2, \\ x_2 &= (M^2_{q\bar{q}} - M^2_V)/W^2 \ll x_1, \end{aligned} \quad (2)$$

see ref. [1]. $M_{q\bar{q}}$ is the mass of the $q\bar{q}$ system produced by a photon of virtuality Q^2. The relevant scale is $\mu^2 = z(1-z)Q^2 + k_T^2 + m_q^2$ where $z, 1-z$ and $\pm\boldsymbol{k}_T$ specify the momenta of the q and $\bar{q}$. The quadratic dependence on g in (1) shows that these data may offer a sensitive constraint on the gluon. Indeed our aim is to show that the off-diagonal distributions are fixed by the conventional diagonal partons, so that the data can, in principle, be included in a global parton analysis.

2. Ji's 'symmetrized' distributions

We shall use the "off-forward" distributions $H(x,\xi) \equiv H(x,\xi,t,\mu^2)$ with support $-1 \leq x \leq 1$ introduced by Ji [2], with the minor difference that the gluon $H_g = xH_g^{\rm Ji}$ [3]. They depend on the momentum fractions

$$x_{1,2} = x \pm \xi \quad (3)$$

carried by the emitted and absorbed partons at each scale μ^2 and on the momentum transfer variable $t = (p-p')^2$. The variables t and ξ do not change as we evolve the distributions up in the scale μ^2. In the limit $\xi \to 0$ they reduce to the conventional parton distributions

$$\begin{aligned} H_q(x,0) &= \begin{cases} q(x) & \text{for} \quad x > 0 \\ -\bar{q}(-x) & \text{for} \quad x < 0, \end{cases} \\ H_g(x,0) &= xg(x), \end{aligned} \quad (4)$$

and satisfy DGLAP evolution. In the limit $\xi \to 1$ they obey ERBL evolution. If we consider H_q at arbitrary values of ξ, then for $x > \xi$ and $x < -\xi$ we have DGLAP-like evolution for quarks and antiquarks respectively, while for $-\xi < x < \xi$ we have ERBL-like evolution for the emitted $q\bar{q}$ pair.

On account of the $x_1 \leftrightarrow x_2$ symmetry the distributions H_q, H_g are symmetric in ξ. We also have symmetry relations in terms of the x variable

$$\begin{aligned} H_q^{NS}(x,\xi) &= H_q^{NS}(-x,\xi), \\ H_q^{S}(x,\xi) &= -H_q^{S}(-x,\xi), \\ H_g(x,\xi) &= H_g(-x,\xi). \end{aligned}$$

where the superscripts S and NS denote singlet and non-singlet quarks respectively.

3. $H(x,\xi)$ in terms of conformal moments

The conformal moments[1] of the off-diagonal distributions,

$$\boxed{O_N(\xi,\mu^2) \;=\; \int_{-1}^{1} dx R_N(x_1,x_2) H(x,\xi),} \qquad (5)$$

are not mixed by evolution

$$O_N(\xi,\mu^2) \;=\; O_N(\xi,\mu_0^2)\left(\frac{\mu^2}{\mu_0^2}\right)^{\gamma_N}, \qquad (6)$$

where γ_N are the same anomalous dimensions as for diagonal partons. The R_N are known polynomials of degree N

$$R_N \;=\; \sum_{k=0}^{N} \left(\begin{array}{c} N \\ k \end{array}\right)\left(\begin{array}{c} N+2p \\ k+p \end{array}\right) x_1^k x_2^{N-k} \qquad (7)$$

with $p=1,2$ for quarks and gluons respectively. The O_N reduce to the usual moments in the limit $\xi \to 0$. For example for quarks

$$O_N \to M_N \;=\; \int_0^1 x^N q(x) dx, \qquad (8)$$

up to a normalizing factor $R_N(1,1)$.

The crucial step is to find the inverse relation to (5). That is to reconstruct $H(x,\xi)$ from a knowledge of the conformal moments. The result, due to Shuvaev [7], is

$$\boxed{H(x,\xi) \;=\; \int_{-1}^{1} dx' K(x,\xi;x') f(x')} \qquad (9)$$

where the kernel K is a known integral [7, 8] and f is the Mellin transform

$$f(x') \;=\; \int \frac{dN}{2\pi i}(x')^{-N} O_N(\xi)/R_N(1,1). \qquad (10)$$

f reduces to the diagonal distribution for $\xi^2 \ll 1$. This follows since [2]

$$\begin{aligned} O_N(\xi) \;&=\; \sum_{k=0}^{[(N+1)/2]} O_{Nk}\xi^{2k} \\ &\simeq\; O_{N0} \;=\; O_N(0) = M_N R_N(1,1) \qquad (11) \end{aligned}$$

for small ξ^2. So the off-diagonal distribution H is completely determined in terms of the diagonal distribution f via (9).

[1]Conformal moments were introduced in [4] for $\xi = 1$, and in [5] for $\xi \neq 1$; see also [6].

4. A good small x,ξ approximation

We can simplify (9) further if we assume that the diagonal partons have the form

$$xq(x) \;=\; N_q x^{-\lambda_q}, \quad xg(x) \;=\; N_g x^{-\lambda_g} \qquad (12)$$

for very small x. Then the x' integration can be performed analytically and

$$H_i(x,\xi) \;=\; \xi^{-\lambda_i - p} F_i\left(\frac{x}{\xi}\right) \qquad (13)$$

with $p=1,0$ for $i=q,g$ respectively. A full set of results for the off-diagonal/diagonal ratios,

$$R_i(x,\xi) \;=\; H_i(x,\xi)/H_i(x+\xi,0), \qquad (14)$$

can be found in [8]. There, the ratios $R_q^{NS,S}$ and R_g are plotted as functions of x/ξ for different values of λ_i. The scale dependence of the off-diagonal distributions, $H_i(x,\xi)$ of (13), and hence of the R_i, is hidden in the μ^2 dependence of the λ_i. Both λ_g and λ_q increase with increasing μ^2.

5. Application to $\gamma^* p \to Vp$

The value of the ratio for the gluon distribution at $x=\xi$ is relevant for diffractive vector meson production, $\gamma^* p \to Vp$, at high energies, see (2). This ratio is given by[2]

$$R_g(x=\xi) \;=\; \frac{2^{2\lambda_g+3}}{\sqrt{\pi}}\frac{\Gamma\left(\lambda_g+\frac{5}{2}\right)}{\Gamma(\lambda_g+4)}. \qquad (15)$$

The cross section formula (1) may then be expressed in terms of the conventional diagonal gluon distribution g,

$$\left.\frac{d\sigma}{dt}(\gamma^* p \to Vp)\right|_{t=0} \;=\; \ldots\left[R_g x_1 g(x_1,\mu^2)\right]^2, (16)$$

where all the off-diagonal effects are contained in the known (enhancement) factor R_g^2. Of course to calculate the cross section properly we must use the unintegrated gluon distribution and integrate over the transverse momenta of the exchanged gluons and of the q and $\bar{q}$ forming the vector meson.

[2]This answer checks with the values of the ratio obtained by direct evolution of the off-diagonal and diagonal gluons in [1].

To obtain the scale dependence of R_g, we first obtain the μ^2 dependence of λ_g of (12) from the behaviour of the gluon found in the global parton analyses. For example, the MRST partons [9] have $\lambda_g = 0.205$ and 0.38 at $\mu^2 = 4$ and 100 GeV^2 respectively. The appropriate scale for the diffractive process $\gamma^*(Q^2)p \to V(q\bar{q})p$ is $\mu^2 \simeq m_q^2 + Q^2/4$. In this way, for diffractive J/ψ and Υ photoproduction at HERA we find that the off-diagonal enhancement, R_g^2, is $(1.15)^2$ and $(1.32)^2$ respectively. However, for Υ photoproduction, x is not sufficiently small (~ 0.01) and we have to improve the assumption made in (12). If we take $xg \sim x^{-\lambda_g}(1-x)^6$ and perform the x' integration in (9) numerically, then we find an enhancement of $(1.41)^2$ for Υ photoproduction [10]. Moreover for ρ electroproduction it is found [11] that the enhancement due to off-diagonal effects of the $\gamma^* p \to \rho p$ cross section $d\sigma/dQ^2$, at the largest Q^2 of the HERA data, is more than a factor 2, which is just the enhancement needed to ensure a perturbative QCD description of the data.

6. Discussion

The main conclusion is embodied in eqs. (9)–(11). That is the skewed distribution $H(x,\xi)$, at any scale, is fully determined at small x,ξ by knowledge of the diagonal parton distribution, at the same scale.

To be sure of this result we have checked that the analytic continuation of the conformal moments O_N in N is allowed [8]. A second consideration is that, from a formal point of view, we may add to the off-diagonal distribution any function which exists only in the ERBL-like region, $|x| < \xi$. In [8] we show such a contribution is negligible $O(\xi^2)$ at small ξ. So far our distributions allow the calculation of the imaginary part of the amplitude for the process. At small x and ξ it turns out that the real part may be calculated easily using a dispersion relation in the c.m. energy squared, W^2, and that the amplitude

$$A \;=\; i\mathrm{Im}A\,\frac{1+e^{-i\pi\lambda}}{1+\cos\pi\lambda}, \tag{17}$$

where $A \propto (W^2)^\lambda$. Finally we note that our result remains valid at NLO, since there is no conformal mixing for $\xi^2 \ll 1$.

We conclude that, at small x,ξ, the skewed distributions $H(x,\xi;\mu^2)$ are completely known in terms of conventional partons. Thus data for processes which are described by such distributions can, in principle, be included in a conventional global analysis to better constrain the low x behaviour of the partons.

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